Analysis of Tai Chi as a Functional Rehabilitation Treatment
for Upper Extremities of Wheelchair Patients

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Abstract

Context: Lack of effective rehabilitation may cause upper extremity atrophy in patients with prolonged periods of wheelchair confinement. Objective: mities by comparing the kinematic and kinetic features of Tai Chi beginning movements between a normal control group and a group of wheelchair patients. Participants: Two groups of subjects participated in this study: the control group (8 healthy male subjects) and the patient group (8 wheelchair-bound male patients with lower extremity injuries). Interventions: The 2-dimensional kinematic images of subjects’ upper extremities are captured while they perform Tai Chi beginning movements. Main Outcome Measures: Two-dimensional kinematic images of the subjects’ upper extremities were captured on high-speed video cameras while the subjects performed the beginning movements of Tai Chi. Computer analysis produced digital data on the movement trajectories and the results were statistically analyzed. Results: Significant differences in the range of motion (ROM) of the upper extremities are captured while they perform Tai Chi beginning movements. Conclusions: By analyzing ROM and joint moments involved in Tai Chi beginning movements, we confirmed that Tai Chi can help wheelchair patients rehabilitate the lost movement of their upper extremities.

Keywords: Patient, Kinematic, Kinetic

Introduction

In long-term wheelchair confinement with lack of effective rehabilitation exercise, the muscles of wheelchair patients tend to atrophy. Other body parts become affected and their functions begin to deteriorate. Further, inadequate exercise contributes to less consumed energy and can subsequently lead to obesity. To combat these conditions, wheelchair patients are advised to resort to medication and physical rehabilitation[1]. When engaging in rehabilitation exercise, they can adopt various types of exercise to achieve the intended amount of muscle workout. In cases where wheelchair patients would like to enhance muscle fitness and control their weight, they can begin by rehabilitating movements in their trunk and upper limbs [2].

In previous studies Tai Chi has been shown to have a beneficial effect on the human body because it is moderate and of low impact [3]. Generally speaking, as joints and vision degenerate, it becomes more and more difficult for a person to maintain their balance. Tai Chi can help increase control over one’s sense of balance; also, it is mentally engaging and rather unlikely to cause great physical injury[4]. Leung and Tsang(2008) reported that wheelchair Tai Chi was very suitable for elderly people who have a poor sense of balance and thus are unable to stand or have difficulty in standing up. It helps them control their movements while incurring less exercise risk[5].

In studies on position balance control, especially those on the prevention of falling, most researchers agree that Tai Chi can improve coordination, enabling practitioners to take protective actions when encountering unexpected circumstances[6-7]. In a 1999 study by Fuller et al.[8], 22 subjects who had difficulty maintaining balance participated in Tai Chi training for 8 weeks. Subsequently, 5 different balance test instruments were used to elucidate the effect of Tai Chi on their sense of balance. It was found that Tai Chi enhanced the patients’ balance. Wong et al. (2008) reported that there was no significant difference in static balance between Tai Chi practitioners and healthy non-practitioners, but with regard to dynamic balance, Tai Chi practitioners performed much better
than non-practitioners. The researchers concluded that Tai Chi practitioners had better coordination and ability to maintain balance [9].

In other clinical research, Tai Chi was incorporated into the medical treatment of osteoarthritis. Lee (2006) found that after doing Tai Chi twice a week in 60-minute sessions for a period of 8 weeks, patients with osteoarthritis were found to have significantly improved in terms of pain relief, balance, and comfort of life [10].

Li et al. (2006) conducted research on the training of muscle strength with Tai Chi and found that subjects who did Tai Chi for 16 weeks significantly strengthened their knee flexors [11]. Taylor-Piliae et al. (2006) reported other studies where elderly people (average age, 66 years) who took 36 Tai Chi courses for 12 weeks felt significant relief from lower back pain and the recovery of muscle strength in their upper extremities [12]. In a study by Qin Choy et al. (2005), doing Tai Chi twice a week in 60-minute sessions for a period of 8 weeks, patients with osteoarthritis were found to have significantly improved in terms of pain relief, balance, and comfort of life [13]. Nakamura et al. (2001) found that Tai Chi could activate more muscles [14], and Christou et al. (2003) found that Tai Chi could improve muscle strength by increasing co-activation of the agonist and antagonist muscles [15]. We therefore based this study on the consistent results achieved by the above studies and adopted Tai Chi to rehabilitate movements in wheelchair patients’ upper extremities.

In the past, researchers designed a variety of models based on the biomechanics of different exercises, including equations applicable to every part of the body [16], equations for certain parts such as legs or arms [17], and the dynamic system for certain joints [18]. Research has also been done on wheelchair movements. For example, Chiu et al. (2007) established an algorithm to compute the energy expenditure of wheelchair patients’ optimal rehabilitation. The wheelchair subject’s body was assumed to be a model with 26 degrees of freedom, and an equation based on these 26 degrees of freedom was established [19]. The parameters for the human body, as proposed by Zatsiorsky and Seluyanov (1983), were adopted to simulate the optimal rehabilitation and compute the resulting energy expenditure [20]. Other researchers used movement analysis data for the purpose of reducing the loading of wheelchair patients’ upper extremities to protect the patients from injury caused by confinement to wheelchairs [21].

To sum up, many studies have been conducted to analyze wheelchair patients’ movements. However, most of them aimed to track patients’ recovery instead of exploring their rehabilitation. Consequently, no concrete evidence can be found to support the hypothesis that inadequate activity has an adverse influence on the upper extremities of wheelchair patients, which motivated us to investigate this topic. Two groups of subjects participated in the study—the control group and the patient group. Photography was used to capture the 2 groups’ beginning movements. Then a comparison is made between the 2 groups. The purpose of this study is to obtain scientific evidence to confirm the supposition that the upper extremities of wheelchair patients are less flexible and their joints are relatively weaker than those of normal subjects. The study also aims to confirm that wheelchair Tai Chi is beneficial to patients and to convince therapists and doctors to prescribe Tai Chi for their patients as a rehabilitation exercise.

**Methods**

**Design**

On the basis of the structure of the arm joints, the human upper extremity model of this study was composed of 5 degrees of freedom (Figure 1). The shoulder joint comprised 3 degrees of freedom, two of which were involved in the translation relative to the x-axis and y-axis, and the remaining one was involved with the rotation of the shoulder joint. The 4th degree involved the rotation of the elbow joint and the 5th degree dealt with the rotation of the wrist joint. In Figure 1, \( l_i \) represents the segment length; \( l_{1i} \), the length of upper arm; \( l_{2i} \), the length of forearm; and \( l_{3i} \), the length of the palm. \( p_i \) represents the translation from the origin of the \( i \)-th link coordinate frame relative to the \( i-1 \)-th link coordinate frame, \( p_i = (x_i, y_i, z_i)^T \), and \( r_i \) represents the position vectors of the center mass of segment \( i \).

![Upper extremity model](image)

**Figure 1. Upper extremity model.**

![Experiment procedures](image)

**Figure 2. Experiment procedures.**

The 5 degrees of freedom served as the variables of this
upper extremity model (Appendix A). Each degree of freedom was assumed to be a coordinate frame. Consequently, according to the Lagrange-Euler equations of motion[22], this model could be illustrated as:

$$\tau_i = \sum_{j=1}^{n} \sum_{k=1}^{m} \text{Trace}(U_{jk} J_{ij} U_{jk}^T) \dot{q}_k + \sum_{j=1}^{n} \sum_{k=1}^{m} \sum_{m=1}^{n} (m_j U_{jk} u_{jm}^T) \dot{q}_m - \sum_{j=1}^{n} (m_j G U_{ji} r_j)$$

$$i = 1, 2, 3, \ldots, 5$$

(Note: All nomenclature is listed at the end of this study.)

The parameters of segments required by the dynamic system included segment length, mass, and the inertia tensor. Zatsiorsky and Seluyanov (1983) adopted gamma ray in their study on human segment parameters[20]. Compared with other studies, the data they established were more precise and complete. Therefore, the segment parameters needed in the dynamic system of this study were based on the data established by Zatsiorsky et al. (1983).

### Table 1. The Range of Motion (ROM) for Upper Extremities

<table>
<thead>
<tr>
<th>Group</th>
<th>Time Interval (s)</th>
<th>Range of Motion (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Shoulder*</td>
</tr>
<tr>
<td>Patient</td>
<td>19.85 ± 1.48</td>
<td>53.66 ± 1.16</td>
</tr>
<tr>
<td>Control</td>
<td>11.06 ± 2.17</td>
<td>66.99 ± 3.70</td>
</tr>
</tbody>
</table>

The ROM for shoulder, elbow, and wrist of the control group and the patient group in wheelchair Tai Chi. *: paired t-test, p < 0.05.

### Table 2. The Peak Angular Velocity for Upper Extremities

<table>
<thead>
<tr>
<th>Group</th>
<th>Time Interval (s)</th>
<th>Peak Angular Velocity (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Shoulder*</td>
</tr>
<tr>
<td>Patient</td>
<td>19.85 ± 1.48</td>
<td>0.127 ± 0.004</td>
</tr>
<tr>
<td>Control</td>
<td>11.06 ± 2.17</td>
<td>0.369 ± 0.027</td>
</tr>
</tbody>
</table>

The peak angular velocity for shoulder, elbow, and wrist of the control group and the patient group in wheelchair Tai Chi. *: paired t-test, p < 0.05.

### Participants

The control group of this study consisted of eight healthy males aged 21 to 24 years who had no history for the past year. Their average height was 173.8 ± 5.2 cm and their average weight was 77.6 ± 7.3 kg. The wheelchair patient group consisted of 8 males aged 26 to 34 years who were unable to stand up due to injury of their lower extremities. They had been confined to wheelchairs for more than 5 years. Their average height was 163.2 ± 2.4 cm and their average weight was 71.8 ± 5.9 kg. All 16 subjects had practiced the beginning movements of Tai Chi for more than 1 month before participating in this study, and they had no history of feeling ill after practicing this exercise.

![Figure 3. Sample of consecutive actions to complete Tai Chi beginning movements. (a) Initial action; (b) final action.](image-url)
Procedures

A high-speed video camera (120 Hz) was set up in front of the subjects to retrieve the 2-D tracks of segment movement. Further, a scale plate was placed behind the subjects that had markers of six anatomic positions of the upper extremities and the trunk: right third phalanges, right styloid process of the ulna, right lateral epicondyle of the humerus, right acromion joint, right temporomandibular joint, and lateral midpoint of trunk. By computer analysis we produced digital data on the movement trajectory of each marked position.

The movements to be analyzed were the beginning movements of Tai Chi of Cheng Man Ching. The movements were completed in the following order: The elbows were raised forward until the arms were at the same height as the shoulder. The hollow of the palms were turned slightly outward. Both arms were raised forward with the elbow adducted and the wrist drooped. The arms were elevated until they were again at the same height as the shoulder. Next, the forearms were tilted and then straightened. Finally, both arms rested naturally on the wheelchair and the hollow of the palms faced backward, relaxed (Figure 3). Each subject was required to perform several rounds of these movements at intervals of 2 min each. Three valid rounds of movements were recorded.

Statistical Analysis

The data collected from the videos were analyzed with a Borland C++ program to transform them into the kinetic parameters needed in this study. We then used paired t-tests to discover any kinematic differences in joints between the 2 groups doing the same Tai Chi beginning movements. The significance level of this comparison was \( \alpha = .05 \).

Results

Figure 4. The shoulder angles on sagittal plane for wheelchair Tai Chi of the control group and patient groups. The two curves represent the average shoulder angle of the control group and the patient group, respectively.

Both groups moved their shoulder joints in a similar way (Figure 4). Both groups showed flexion at first and slight extension at the final stage of the movements. The most significant difference lay in the angle of flexion. The peak value of the flexion angle for the control group could be up to 60\(^\circ\), while the peak value for the patient group was 40 \(^\circ\). Throughout these movements, the control group had significantly greater ROM than the patient group.

Figure 5. The elbow angles on sagittal plane for wheelchair Tai Chi of the control group and patient groups. The two curves represent the average elbow angle of the control group and the patient group, respectively.

Figure 6. The wrist angles on sagittal plane for wheelchair Tai Chi of the control group and patient groups. The two curves represent the average wrist angle of the control group and the patient group, respectively.

Figure 5 indicates that both groups moved their elbows in a similar way. They all showed relative flexion of the elbows. Additionally, 2 peak values for the relative flexion were found, which occurred when subjects raised their elbow and adducted
it. However, the control group had greater ROM than the patient group (Table 2).

Figure 6 presents the activity of the wrist joint on the sagittal plane. Although there were similarities in the movement of shoulder joints and elbows, this was not the case in the movement of the wrist joints. Since movement of the wrist joint is relatively complex, we found significant individual differences. Nevertheless, the control group had greater ROM of wrist joints than the patient group, especially in the final stage of the Tai Chi movements. In that stage the patient group showed no relative flexion, possibly because of less ROM. In contrast, the control group demonstrated greater flexion.

To further investigate the effect of muscle strength on the upper extremities, we computed the joint moments and compared them in the 2 groups. Figure 7 illustrates the moments produced by the shoulders; the 2 groups moved in a similar way. The peak value of the moments for the control group reached 13 Nm, whereas the peak value for the patient group was approximately 7 Nm.

Figure 8 presents the elbow moments produced by the 2 groups. The elbow moments for the control group ranged from 2.5 to 3.5 Nm, whereas the elbow moments ranged from 2 to 2.5 Nm for the patient group.

Figure 9 indicates the wrist moments for the 2 groups. Like the results of the elbow moments, the control group showed larger wrist moments, ranging from 0.2 Nm to 0.75 Nm, whereas the wrist moments for the patient group ranged from 0.1 Nm to 0.23 Nm.

ROM is the clinical criterion used to judge whether patients recover normal actual movements. Therefore, we recorded the time each group spent in completing the Tai Chi beginning movements as well as the ROM of shoulder, elbow, and wrist joints. The peak value and the minimal value for a given joint would determine its ROM.

Table 1 summarizes the statistics used to quantify the differences in the 3 joints between the 2 groups. The two groups were differentiated in shoulder and wrist, especially in the ROM of the wrist, but there were no statistically significant differences in elbow ROM.

The maximal angular velocities for each of the three segments are presented in Table 2. The two groups showed significant differences in maximal angular velocity: on average, the peak value for each angular velocity of the patient group was 0.2 rad/s less than that of the control group. These results confirmed that the wheelchair patients were less competent in exercising their upper extremities. The phenomenon may result from the failure to engage in more sports.

Table 3. The Peak Moment for Upper Extremities

<table>
<thead>
<tr>
<th>Group</th>
<th>Time Interval (s)</th>
<th>Shoulder*</th>
<th>Elbow*</th>
<th>Wrist*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patient</td>
<td>19.85 ± 1.48</td>
<td>7.305 ± 0.397</td>
<td>2.428 ± 0.277</td>
<td>0.240 ± 0.032</td>
</tr>
<tr>
<td>Control</td>
<td>11.06 ± 2.17</td>
<td>12.137 ± 0.885</td>
<td>3.608 ± 0.240</td>
<td>0.456 ± 0.034</td>
</tr>
</tbody>
</table>

The peak moment for shoulder, elbow, and wrist of the control group and the patient group in wheelchair Tai Chi. *: paired t-test, p < 0.05.
Table 3 presents the optimal moment of each segment. The shoulders, elbows, and wrists of the patient group produced smaller moments than the control group. Particularly, the shoulder moments of the patient group averaged 5 Nm, significantly smaller ($p < 0.05$) than in the control group.

**Discussion**

Previous studies proved that practicing Tai Chi can lead to such benefits as the enhancement of muscle strength, control over posture, and flexibility. These studies formed the basis of our attempt to confirm the efficacy of Tai Chi as rehabilitation exercise for wheelchair patients. Our experimental results can also provide guidance on how to improve wheelchair patients’ movement to increase rehabilitation efficacy.

**ROM**

We computed the ROM for each of the three segments and found significant differences between the two groups in the shoulder ROM and wrist ROM (Table 1). The control group had greater shoulder ROM than the patient group (Figure 3). In spite of this, it is believed that wheelchair patients can restore their original and better shoulder ROM as long as they practice Tai Chi properly. On the other hand, the control group had greater wrist ROM than the patient group (Figure 5). This result was different from what we expected. Possible reasons could be that the wrist was used a lot in the Tai Chi beginning movements, and the wrist action was more complicated. Before the experiment began, the subjects had been corrected on their posture when practicing the movements for 30 minutes; however, this was unlikely to eliminate individual wrist ROM differences. This phenomenon was not unique to this study. Xu et al. (2003) reported that a shift of center of mass (COM), accompanied by a change of Tai Chi movement, resulted in greater joint ROM [23]. Consequently, it was not surprising that a comparatively dramatic change of wrist ROM occurred in this study.

Regarding elbow ROM, no significant difference was found between the two groups (Figure 4). This result corresponded to findings by Monteiro et al. (2008), who studied female subjects confined to wheelchairs. They discovered that when their subjects engaged in muscle strength and flexibility workouts, the horizontally adducted shoulder muscle was strengthened. However, the workouts did not have the same effect on the ROM of elbow flexion [24].

According to the above discussion on ROM, we conclude that the notion proposed by Sauers et al. (2007) is feasible. They found that appropriate passive shoulder stretching could increase the mobility of pitching and therefore suggested that wheelchair patients stretch as much as possible and keep the movement pace slow. In this way, muscle coordination would be enhanced, leading to the recovery of ROM for joints. Slow pace of movement is exactly what Tai Chi requires of its practitioners, and so their notion could be considered to support the intention of this study to have wheelchair patients enlarge ROM and strengthen muscles by practicing Tai Chi.

**Moment**

Tai Chi also plays an important part in muscle dynamics since it can stimulate muscle and cutaneous receptors. Receptors give feedback to joint loading, conducted to the central nervous system through sensory nerves, to bring about various types of motions. Tai Chi can help improve neuromuscular control because it stimulates muscles and cutaneous receptors, which helps wheelchair patients strengthen their muscles and increase joint moments.

We compared the moments for upper extremities (Figures 7, 8, 9) and found significant differences in the optimal moments between the two groups (Table 3). The differences arose from the fact that the wheelchair patients had spent little time doing exercise for a long period and their muscle strength had degenerated. This finding is similar to what Niehoff et al. (2004) documented in their study of one subject group confined to wheelchairs and another group that had the habit of taking exercise. After the experiment, they found that the exercise group had better body composition than the group confined to wheelchairs [26].

In our study, the wheelchair patient group took more time to complete the Tai Chi movements than the control group (Table 1), but this was not the factor that caused the differences in joint moment since all the data had been normalized. Despite our effort to make the two groups complete the movements in the same amount of time, it was difficult to achieve so. In view of this, the focus of this study was only on the precision of movement in order to achieve the goal of effective exercise. Lumsden et al. (1998) found that Tai Chi worked well as physical therapy on patients suffering from osteoarthritis. They concluded that Tai Chi could improve the ROM of upper extremities, help relieve the pain of shoulder joints, and enhance flexibility and muscle strength[27]. Therefore, we consider Tai Chi to be an excellent method of rehabilitation for wheelchair patients, and we have demonstrated that Tai Chi can increase the ROM of the three segments of upper extremities, including the maximal stretching and maximal flexion. However, it is important to rehabilitate at a low speed of movement.

Horak (1991) proposed the theory of posture control [28], which dealt with muscle skeleton, the neuromuscular system, and the strategies of sensation. Applying Horak’s theory to wheelchair patient rehabilitation, a physical therapist can impose appropriate resistance on patients who are engaged in rehabilitation. When posture is controlled, the upper extremity moments can be restored to normal.

Synthesis of the above discussion suggests that exploring ROM and moments can really give us a whole picture of how rehabilitation works and can further enhance its efficacy. The recovery of ROM is the top priority for rehabilitating wheelchair patients’ upper extremities. The ROM of the control group can function as the potential target for wheelchair patients stretching their upper extremities, and we recommend wheelchair Tai Chi practitioners to stretch the upper extremities and prevent possible injury. As for joint moments, the focus should be on the restoration of muscular strength, which refers to the skill and capability of muscles in storing and reusing energy. We suggest that patients be given...
some amount of drag force when they do the Tai Chi beginning movements. Meanwhile, the training should be intensified progressively, especially in the hand-pushing and hand-drawing-back process. When patients do movements involving isotonic contraction and concentric contraction, it can activate muscle nerves to perform the muscle’s dominating function, thus awake more contractible muscles to go into the exercise, and enhance the mechanical efficacy of Tai Chi.

Conclusions
Evaluating and analyzing Tai Chi in terms of kinetics and kinematics, we found that Tai Chi can effectively rehabilitate the upper extremities of wheelchair patients. We used 2-D analysis to explore the features of upper extremities during wheelchair Tai Chi. We found significant differences in joint ROM and moments between the control group and the patient group. To improve joint ROM, we recommend that wheelchair patients stretch as much as possible and keep their Tai Chi movements slow. The smaller joint moments of wheelchair patients might be connected with the degeneration of muscle strength; to recover their muscle strength, wheelchair patients should be given proper loading when they do Tai Chi. In summary, the above kinetic and kinematic analysis of wheelchair Tai Chi demonstrates that Tai Chi is helpful to rehabilitate the upper extremities of wheelchair patients.

Appendix A

Chiu et al. defined seven types of homogeneous transformation matrices (CH-7T) [29-32]. Our study adopted three of those seven types to design the Lagrange-Euler equations. For example, in the basic homogeneous rotation matrix of Eqs. 2 through 4, the symbol $q_i$ represents the generalized coordinate, and $c_{q_i} = \cos q_i$, $s_{q_i} = \sin q_i$. The translation from the origin of the $i$th link coordinate frame relative to the $i$-th link coordinate frame is represented with $p_i$, and $p_i = (x_i, y_i, 0)$. $E_i$ is the homogeneous transformation matrix of the $i$-th link coordinate frame relative to the $i$-th link coordinate frame.

$$E_i = \begin{bmatrix} 1 & 0 & 0 & x_i \\ 0 & 1 & 0 & y_i \\ 0 & 0 & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 4 Homogeneous transformation matrices and parameters for the arm model

<table>
<thead>
<tr>
<th>variable</th>
<th>$i\rightarrow i+1$</th>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$z_i$</th>
<th>$0E_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>$E_1 = E_1^{(s)}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$0E_1 = E_1^{(s)}$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$E_2 = E_2^{(s)}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$0E_2 = E_1^{(s)}E_2^{(s)}$</td>
</tr>
<tr>
<td>$q_3$</td>
<td>$E_3 = E_3^{(8)}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$0E_3 = E_1^{(s)}E_2^{(s)}E_3^{(8)}$</td>
</tr>
<tr>
<td>$q_4$</td>
<td>$E_4 = E_4^{(k)}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$0E_4 = E_1^{(s)}E_2^{(s)}E_3^{(8)}E_4^{(k)}$</td>
</tr>
<tr>
<td>$q_5$</td>
<td>$E_5 = E_5^{(8)}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$0E_5 = E_1^{(s)}E_2^{(s)}E_3^{(8)}E_4^{(8)}E_5^{(8)}$</td>
</tr>
</tbody>
</table>

Note: "•" meant $i\rightarrow iE_i$ had no position vector $p_i$.

Appendix B

Chiu et al. defined seven types of homogeneous transformation matrices (CH-7T) [29-32]. Our study adopted three of those seven types to design the Lagrange-Euler equations. For example, in the basic homogeneous rotation matrix of Eqs. 2 through 4, the symbol $q_i$ represents the generalized coordinate, and $c_{q_i} = \cos q_i$, $s_{q_i} = \sin q_i$. The translation from the origin of the $i$th link coordinate frame relative to the $i$-th link coordinate frame is represented with $p_i$, and $p_i = (x_i, y_i, 0)$. $E_i$ is the homogeneous transformation matrix of the $i$-th link coordinate frame relative to the $i$-th link coordinate frame.

$$\ddot{E}_i = \text{generalized acceleration}$$

$E_i = \text{the homogeneous transformation matrix of the } i\text{-th coordinate frame relative to the } i\text{-th coordinate frame}$

$0E_i = \text{the coordinate transformation matrix from 0 coordinate frame to the } i\text{-th coordinate frame, and the}$

$\dot{U}_{ij} = \text{was defined as } \dot{E}_i \dot{E}_j$ (see Appendix B)

$\dot{U}_{ijk} = \text{was defined as } \dot{E}_i \dot{E}_j \dot{E}_k$ (see Appendix B)

$J_i = \text{pseudo-inertia matrix} \quad 0E_i = \text{the coordinate transformation matrix from 0 coordinate frame to the } i\text{-th coordinate frame, and the}$

$\dot{E}_i = \text{the homogeneous transformation matrix of the } i\text{-th coordinate frame relative to the } i\text{-th coordinate frame}$

$G = [0,0,-g,0], \quad g = 9.8062 \text{ ms}^{-2}$

$\dot{\mathbf{r}_i} = (\mathbf{\bar{r}_i}, \mathbf{\bar{r}_i}), \quad \mathbf{\bar{r}_i} = \text{position of the center of mass (COM) for}$

$\text{jth link}$

$\mathbf{\bar{r}_i} = \text{position of the center of mass (COM) for}$

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