Mathematical Model for the Optimal Tennis Placement and Defense Space

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Abstract

The purpose of the study is to formulate the offense and defense model in a tennis match by adopting mathematical methods to locate tennis placements and the defense space. A four-phase study was designed. First, the flying algorithm was established with the application of physics theory. Next, with the use of Spline function, the algorithm for 3D defense space of the receiver was formulated. Then two male students in a college tennis team (Age: 21±1.9 years; Height: 175.5±3.5cm; weight: 70.0±1.4Kg) were asked to move from the preparatory position to hit each of the 100 balls evenly distributed in the space. The data on each returning time were collected through the high-speed video camera (200Hz). In the fourth phase, based on the established algorithm, a computer program was edited to simulate serving and service returning in an attempt to locate the optimal tennis placement and the best defense space. The result showed that: (1) Whether in singles or doubles, a better placement by the striker would be achieved if the service was farther away from the two sides of the receiver (V effect), (2) In singles, if the initial position of the receiver and the contact position of the striker form a diagonal line or a bisecting line, it would lead to the optimal defense space (area bisecting effect), (3) In doubles, if the contact position of the striker and the preparatory positions of two receivers formed an isosceles triangle, it would be the best defense space for the receivers (isosceles triangle effect). When the above three effects are further verified through real-life tennis matches, the designed algorithm can be used to establish an offense and defense model of multiballs; moreover, the simulation model can assist tennis players in effective training.

Keywords: Defense space, Algorithm, Area bisecting effect, Isosceles triangle effect

Introduction

To win a match, tennis players are expected to possess comprehensive skills, and adopt aggressive and flexible strategies based on the type of course and the rival’s capability. Strikers try to score by having the optimal placement or making the return to their advantage. On the other hand, receivers attempt to predict the advantageous defense position, reduce errors, or make the return to their advantage [1].

Literature Review: A wide range of studies on tennis were conducted in the past. For example, in motion analysis of tennis stroke, Akutagawa & Kojima [2] once explored the effect on the contact speed caused by torsional moment, which was formed by the player’s trunk and hip joints. Tanabe & Ito [3] used to analyze the ball contact during tennis serving with 3-D camera. The result demonstrated that the contact speed is determined by the angle range and angular velocity of the shoulder joint. Tasi & Ito [4] once conducted a service test on four tennis champions of National Taiwan University. It was found that their highest contact speed ranged from 50m/sec to 60m/sec.

Steele et al. [5] used to conduct a related experiment on the material made into the tennis ball. First, he classified the balls into four levels according to the degree of being worn out. Then he conducted a wind tunnel experiment on the balls. It was found that the drag coefficient varied in different phases and that drag coefficient on the surface of the new ball ranged from 0.7 to 0.9.

Wang [6] once tested the ball’s coefficient of restitution (CR), and obtained the value of 0.69 in the red clay court. The value approximated that found in the experiment by Wong [7]. On the other hand, Haake et al. [8] conducted an experiment on the impact of the ball on the racket. It was found CR would decline with the increase of the impact velocity. The possible reason for it was the vibration dampening caused by ant-shock from grip and racket strings of the racket.

As to the computer simulation, Chiu [9] adopted numerical algorithm to simulate the flight of the table tennis, volleyball, soccer ball, etc. The result indicated that the error incurred in predicting the distance of flight would be tiny, if the time step of calculation was set to be 0.0001sec. Moreover, Chiu [10] once applied aerodynamics and numerical methods to design the defense and offense system for flat service and service return of tennis. It was found that this simulation system was capable of projecting the optimal placement position of the flat service. To meet the demands of the domestic badminton players, Chiu [1] also attempted to construct the algorithm for
defense and offense in badminton singles, which proved to accurately locate the optimal placement position and the best defense position [1].

In tennis competitions, one of the players’ concerns is to locate the optimal placement. Another concern is to locate the optimal defense space so that fewer errors will occur and a better chance of winning will be achieved. In view of this, this study aimed to construct the defense and offense model, which involves numerical methods and computer simulation to facilitate the prediction of the tennis flight. The model can help train players prior to the contests, assisting them in locating the optimal placement position and the most advantageous defense position.

**Methods**

**Subjects**

The participants were two male players of NCHU school tennis team (age: 21±1.9 years; height: 175.5±3.5cm; weight: 70.0±1.4Kg), who were right-handed and had more than three years of experiences in competitions. When they stood holding the racket, the highest contact point was 276cm ±2.1. No injuries were reported for the past six months. Plus, they showed no signs of sore or pain when engaging in this experiment.

**Mathematical Model**

A Cartesian coordinate system OXYZ (Fig. 1a) was assumed to be in the middle of the court. The striker was on the left side of the court, and the receiver was on the right side. \( t_k \) represented a certain time point during the consecutive ball flight. The time point for the contact was assumed to be \( t_0 \) when the receiver moved from the initial position to return the service. If the receiver could return the ball before \( t_{k+1} \), the moment it touched the ground the second time, then the return would be successful. If the receiver failed to return the service, the striker would score one point.

**Flying algorithm for the tennis ball**

The position vector \( P_0 \) of the Cartesian coordinate system OXYZ served as the origin of the translation coordinate system \( OX'Y'Z' \) (Fig. 1b), whose coordinate origin was defined as the striker’s center of mass (COM) at \( t_0 \). The direction angle of the service was represented with \( \alpha \) and its angle of elevation was \( \beta \). \( V_0 \) represented the ball’s initial velocity vector relative to air (Fig. 1c).

In the course of flight from the contact to the second landing (Fig.1a), \( P_k = [X_k, Y_k, Z_k]^T \) represented the position vector of the ball in \( t_k \). As a consequence, the horizontal distance from the initial position vector \( P_0 = [X_0, Y_0, Z_0]^T \) to \( P_k = [X_k, Y_k, Z_k]^T \) in \( t_k \) would be illustrated as:

\[
d = [(X_k - X_0)^2 + (Y_k - Y_0)^2]^{1/2}.
\]

In the initial position vector \( P_0 = [X_0, Y_0, Z_0]^T \), \( X_0, Y_0 \) represented plane XY in the Cartesian coordinate system OXYZ and \( Z_0 \) referred to the contact height. In the course of flight, two neighboring time points \( t_k \) and \( t_{k+1} \) were selected at random, and the interval between them was...
\[ \Delta t = t_{k+1} - t_k \] When \( t_k \) approximated \( t_{k+1} \), the movement of the ball’s COM from \( t_k \) to \( t_{k+1} \) could be perceived as a constant acceleration motion \[9-14\]. Thus, the position vector \( P_{k+1} \) in time \( t_{k+1} \) could be computed with equation (1):

\[ P_{k+1} = P_k + V_k \Delta t + \frac{1}{2} A_k \Delta t^2 \] \( (1) \)

In the above equation, \( V_k = [V_{x_k}, V_{y_k}, V_{z_k}]^T \) and \( A_k = [A_{x_k}, A_{y_k}, A_{z_k}]^T \) respectively referred to the velocity and the acceleration vector of the ball’s COM. \( V_{k+1} \) in time point \( t_{k+1} \) could be computed with equation (2) and \( A_k \) in time point \( t_k \) could be computed with equation (3):

\[ V_{k+1} = V_k + A_k \Delta t \] \( (2) \)

\[ A_k = F_i/m \] \( (3) \)

\( F_i \) represented the external force exerted on the ball in flight, \( m \) referred to the ball’s mass. \( V_0 \) represented the initial velocity vector for the ball’s mass on the contact, and \( V_0 = A_0 \alpha \beta ([V_{x_0}, V_{y_0}, V_{z_0}]^T \) (Fig 1b). \( V_0 \) represented the contact speed. \( A_0 \alpha \beta \) referred to the rotation coordinate transformation matrices:

\[ A_0 \alpha \beta = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 & 0 & \cos \beta & -\sin \beta & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \] \( (4) \)

The flying tennis takes on three forces, namely the gravity, drag, and lift (Fig 2b). Since this study focused on the flat service, which involved no spin, the lift was considered to be zero. Therefore, \( F_i \) in the time point \( t_k \) could be illustrated as:

\[ F_i = \frac{1}{2} \rho C_d A_d V_k^2 u_k + mg \] \( (5) \)

In equation (5), \( \rho \) represented the air density, which measured 1.23kg/m³ in the standard atmosphere pressure[15]. \( A_d \) represented the drag area of the tennis (\( \pi r^2 \) \( r \) meant the radius of the tennis). \( C_d \) referred to the drag coefficient, and \( C_d = 0.85[5] \). \( G \) represented the acceleration vector of gravity, and \( G = [0, 0, -g]^T \). \( g \) referred to acceleration of gravity, and \( g = 9.81 \) m/s². \( V_k \) represented the velocity of the tennis relative to air in the time point \( t_k \). \( u_k \) referred to the unit vector of the drag in time point \( t_k \), and \( u_k = V_k/|V_k| \). The velocity vector relative to air was \( V_0 \) (Fig 1b); therefore, the relative velocity \( V_{r0} = |V_{0r}| \).

In this study, the position vector of the ball’s COM, \( P_0 = [X_0, Y_0, Z_0]^T \), was used to judge the variation in velocity after the ball’s first impact on the ground (the time point for the first impact was assumed to be \( t_1 \)). Suppose the ball touched the ground in \( t_k \), the velocity vector could be written as \( V_k = [V_{x_0}, V_{y_0}, V_{z_0}]^T \). \( Z_0 \) represented the coordinate value along the Z axis. If on impact, \( Z_0 \leq 0 \), it meant the direction of velocity components for \( V_k \) on Z axis was opposite to its original direction, so the velocity of \( V_k \) was equivalent to the original velocity components multiplied by the coefficient of restitution \( e \).

**Requirements for successful flight over the net**

The ball’s COM in the consecutive flight was represented by \( P_{k+1} \), and \( P_0 = [X_0, Y_0, Z_0]^T \). The position vector for the ball’s impact on the ground was assumed to be \( [X_0, Y_0, Z_0 - r]^T \). The requirements for successful flight over the net in time point \( t_k \) are:

\[ |X_k| \leq 0.05m, |Y_k| \leq 4.115m, \text{ and } Z_k - r \geq H_{net} \]

\((1) \)

According to the regulations set by the International Tennis Federation (ITF), the radius of the tennis \( r \) is 0.032m, the distance between either side line and the central line \( Y_0 \) is 4.115m, and the height of the net \( H_{net} \) is 0.914m.

**Requirements for a valid placement**

A target placement position vector was defined as \( M = [X, Y, 0]^T \). The position vector in \( t_1 \), the time point for the first ground touching, was set as \( P_1 = [X_1, Y_1, Z_1]^T \). With the two position vectors, \( P_1 = [X_1, Y_1, Z_1]^T \) and \( P_k \), the direction angle of the service could be computed with equation (6):

\[ a = \tan^{-1} \left( \frac{Y - Y_0}{X - X_0} \right) \] \( (6) \)

To judge if the first ground touching fell inside the presumed range, the flying distance had to be calculated. Suppose the angle of elevation was \( \beta \), and \( \beta = \beta_{min} + m \times \Delta \theta \) (\( m=1,2,..,n \); \( \Delta \theta = (\beta_{max} - \beta_{min})/n \), \( n \) was integral; \( \Delta \theta \) : angle of step). Values ranging from the minimal one \( \beta_{min} \) to the maximal one \( \beta_{max} \) were input into the equation for tennis flight (Eq.7) to estimate the distance and specify \( \beta \) for the target area. It was hypothesized that the position vector for the ball’s impact on the ground was \( [X_k, Y_k, Z_k - r]^T \). In \( t_k \), the time point for the first ground touching, the horizontal distance between \( P_k \) and \( L \) (\( L \) represented the target placement) could be computed by using equation (7):

\[ ds = [(X_k - X)^2 + (Y_k - Y)^2]^{1/2} \]

A tolerated error of 0.05m was presumed for \( ds \) (Fig 1a). As a result, the conditions for valid first ground touching could be written as \( Z_k - r \leq 0 \) and \( |ds| \leq 0.05m \).

\[ ds = [(X_k - X)^2 + (Y_k - Y)^2]^{1/2} \]

**Algorithm for the valid defense space**

The defense space in this study referred to the maximal space the service could be returned before it touched the ground the second time. In the center of the tennis court was
the origin of the Cartesian coordinate system \(OXYZ\) (Figure 1). On the left side of the court was the striker, whose service contact was assumed to be the origin of the translation coordinate system \(O'X'Y'Z'\). On the right side of the court was the receiver, who got ready on the origin of the cylindrical coordinate system \(OXsYsZs\) to return the service (Fig.1,3). The calculation steps involved in the entire process were elaborated here:

Step 1: The flying equation was adopted to calculate the position vector \(\mathbf{P}_1\) relative to the Cartesian coordinate system \(OXYZ\) in \(t_k\) (Fig. 1a; Fig. 1c). Then \(\mathbf{P}_1\) was coordinately transformed into \(R(\mathbf{R}=\{r_1, \theta_1, h_1\})\), the position vector of the cylindrical coordinate system \(OXsYsZs\) (\(r_1\): radius; \(\theta_1\): azimuth; \(h_1\): height).

Step 2: A defense space for returning the service was established based on the origin of the cylindrical coordinate system \(OXsYsZs\), also the preparatory position of the receiver (Fig.2a, 2b). The defense space constituted four planes \(A, B, C, D\) which parallel \(OXsYs\) plane. The height of each plane in the cylindrical coordinate system \(OXsYsZs\) was indicated by \(h_1, h_2, h_3, h_4\). Each plane had 25 contact points (Fig.2, Fig.3a), so four planes had totally 100 contact points.

Step 3: Take plane A as an example (Fig. 2a). There were 25 contact points on plane A (Fig. 2c). One of the contact points was the origin. The other 24 contact points distributed evenly in eight direction axes—east (E), west (W), south (S), north (N), northwest (NW), southwest (SW), southeast (SE), and northeast (NE). Namely, each direction axis comprised three contact points and the origin \(r_0\). The 24 contact points formed three concentric circles \(S_1, S_2, S_3\), whose radius was \(r_0, r_2, r_3\).

Step 4: The movement of the receiver along the \(Xs\) axis of plane A was elaborated here (Figure 2). Supposed the time the receiver spent in moving from the origin of the cylindrical coordinate system \(OXsYsZs\) to defense points \(r_0, r_1, r_2, r_3\) \((R=\{r_0, r_1, r_2, r_3\})\) was \(t^{(k)}_1, t^{(k)}_2, t^{(k)}_3, t^{(k)}_4\) (Spine interpolation was applied to the collected data on \(r\) and \(t^{(k)}\) to construct a set of spline functions. In this way, the time consumed in moving from the origin to \(r_k\) along the \(Xs\) axis of plane A could be calculated with equation (8): \(t_k^{(k)}=\text{Spline}(r_k^{(k)}, r_k)\) (8)

Step 5: Following step 4, the time the receiver spent in moving from the origin of cylindrical coordinate system \(OXsYsZs\) (Fig. 2b) to \(r_k\) on any of the eight direction axes was assumed to be \(t_k^{(k)}=\{t_k^{(k)}1,t_k^{(k)2},t_k^{(k)3},t_k^{(k)4},t_k^{(k)5},t_k^{(k)6},t_k^{(k)7},t_k^{(k)8}\}\) (9)

Step 6: The height of the planes A, B, C, D was represented with \(h_1, h_2, h_3, h_4\) \((H=[h_1, h_2, h_3, h_4])\) (Fig.4c). On each plane, the receiver was asked to move from the origin of the cylindrical coordinate system \(OXsYsZs\) at the azimuth angle of \(\theta_k\) to \(r_k\) and the time consumed was assumed to be \(t_{\theta_1}^{(k)}=\{t_{\theta_1}^{(k)}1,t_{\theta_1}^{(k)2},t_{\theta_1}^{(k)3},t_{\theta_1}^{(k)4},t_{\theta_1}^{(k)5},t_{\theta_1}^{(k)6},t_{\theta_1}^{(k)7},t_{\theta_1}^{(k)8}\}\) (9)

Step 7: The receiver moved from the origin to \(R(\mathbf{R}=\{\theta_k, r_0, h_1\})\) to return the service, and the time consumed was represented with \(t_{\theta_0}^{(k)}\). When the ball flew to the position vector \(P_k=[X_k, Y_k, Z_k]^{T}(t_k, k=0,1,\ldots,n)\), \(t_{\theta_0}^{(k)}\) could be calculated by using steps 1 to 6. If \(t_{\theta_0}^{(k)}\) was longer than \(t_k\), the time point the ball touched the ground for the second time,
it meant that the receiver couldn’t intercept the ball and the striker would gain one point.

The optimal placement
In singles, the court on the receiver’s part was divided into 100 rectangles of the same size. The center of each rectangle was defined as the target placement of the service. Next, the flying algorithm and the defense algorithm were adopted to calculate the time the receiver spent in moving from the preparatory position to return each service. The computer sequenced the returns in the order of the consumed time and generated a priority of service returning. The service which took the longest time to return would lead to the optimal placement.

The optimal defense space
In doubles, the court on the receivers’ part was divided into 25 rectangles of the same size. The center of each rectangle was defined as the target placement of the service. Next, the center of each rectangle was assumed to be the receiver’s preparatory position. The computer sequenced the returns in order of the consumed time and generated a priority of service returning. The service which took the longest time to return would lead to the optimal placement.

Data Collection Procedure
The collected data came from the picture taking of two subjects’ returning the service with two high-speed video cameras (200Hz). The origin of the coordinate system \( OXsYsZs \) served as the center of circle \( r_0 \) \((r_0=0m) \) for three concentric circles \( S_1, S_2, S_3 \), whose radius was \( r_0, r_2, r_3 \) \((r_1=1.9m \times r_2=5.6m \times r_3=9.6m) \). A direction light plate was placed 25m north of \( r_0 \). On either side of \( r_0 \) was a video camera, 25m away. The cameras could shoot the subjects’ movement and the signals of the direction light plate. The subject stood on the origin \( r_0 \), the preparatory position (Fig 3a, 3b), with his heels separated for 0.5m. The heels were 0.5m away from \( Ys \) axis, and an isosceles triangle was formed by the origin of \( OXsYsZs \) and the two heels (Fig. 3b). On the direction light plate, each of the eight directions had an indication light (Figure 3c). The experimenter randomly turned on one of the direction lights and recorded the time the subjects spent in returning the service.

![Diagram](image)

**Figure 3.** (a) the setting of experiment (b) the direction light plate (c) the preparatory position of the receiver

First, on circle \( S_3 \), each of the eight direction axes E, W, S, N, NW, SW, SE, NE had a plumb line (Fig. 3), to which four tennis balls named a, b, c, and d were attached at different heights \((h_1=0.15m, h_2=0.90m, h_1=1.70m, h_3=2.75m)\). Then the experimenter randomly pressed one of the eight direction lights (each direction light could only be pressed once). Seeing the light, the subject had to move from the preparatory position to the indicated direction to hit ball a. Balls labeled a of the eight directions were hit one by one in this way and each movement time was recorded. Subsequently, the movement time in hitting balls b, c, d in each direction was recorded. The above procedure was applied to circles \( S_2 \) and \( S_1 \) to record the movement time spent in hitting balls a, b, c, d of eight directions. Finally, the time consumed in hitting four balls a, b,
c, d on S₀ were recorded, too. In sum, 100 balls were hitting. Two rounds were conducted; that is, each ball was hit two times, and the hitting which took less time was chosen as the test value. The subjects hit the balls at the interval of 90 seconds.

**Parameters**

The researcher of this study edited software in computer programming language Borland C++ to simulate the tennis flight and to test the flying algorithm and defense algorithm. Four physics parameters had to be confirmed before the simulation and testing. One of them was the drag coefficient. As was indicated in the graph of the tennis' drag coefficient by Steele et al [5], the coefficient $C_d$ was 0.78. Another parameter was the coefficient of restitution. According to the study by Wong [7], on the red clay court, the coefficient of restitution for flat service with no spin was 0.69. Still another parameter was the time step for the simulation of the flying tennis. The time step was assumed to be $\Delta t \leq 0.0001 \text{sec}$ in this study. The reason was that the flying horizontal distance would remain the same (Table 1) and the relative error $Re$ was below 0.002%. The last parameter was angle step $\Delta \theta$ of $\beta$ in locating the tennis placement. The testing result showed that it was preferable to take $\Delta \theta = 0.01^\circ$ (Table 2), since the incurred error $Re$ for adopting $\Delta \theta = 0.01^\circ$ to predict the placement was 21.68% (<100%), which was within the acceptable range $|ds|$.

**Table 1. The numerical time step $\Delta t$ for tennis flying algorithm**

<table>
<thead>
<tr>
<th>$\Delta t$ (sec)</th>
<th>Tf (sec)</th>
<th>Distance(m)</th>
<th>Re(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.84000</td>
<td>25.8211</td>
<td>0.091</td>
</tr>
<tr>
<td>0.01</td>
<td>0.84000</td>
<td>25.8933</td>
<td>0.371</td>
</tr>
<tr>
<td>0.001</td>
<td>0.83299</td>
<td>25.8029</td>
<td>0.021</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.83253</td>
<td>25.7982</td>
<td>0.002</td>
</tr>
<tr>
<td>0.00001</td>
<td>0.83320</td>
<td>25.7976</td>
<td></td>
</tr>
</tbody>
</table>

Note: $Vs=45\text{m/s}$, $\alpha=0$, $Z_0=2.73\text{m}$; $Re=100\% \times (\text{distance}-25.7976)/25.7976$

**Table 2. The error for adopting $\Delta \theta$ to predict the placement**

| $\Delta \theta$ (deg) | $\theta$ (deg) | $|ds|$ (m) | Re(%) |
|------------------------|----------------|-----------|-------|
| 0.2                    | -4.60          | 0.05868   | 117.36|
| 0.1                    | -4.60          | 0.03559   | 71.18 |
| 0.05                   | -4.58          | 0.02759   | 55.18 |
| 0.01                   | -4.57          | 0.01623   | 32.46 |
| 0.005                  | -4.57          | 0.01084   | 21.68 |

Note: $\Delta t = 0.001 \text{sec}$; $Vs=45\text{m/s}$; $\alpha=0$; contact position (0,0,2.73); placement (12, 0,0); $Re=100\% \times |ds|/0.05$; $\theta$: the estimated angle of elevation.

**Result and Discussion**

**Defense space**

Figure 4. a,b,c indicated the defense space for subject P at the movement time of 0.7s, 1.0s, 1.3s; d,e,f indicated the defense space for subject Q at the movement time of 0.7s, 1.0s, 1.3s (See Table 3)
Table 3. The defense space for subjects P and Q

<table>
<thead>
<tr>
<th>Subjects</th>
<th>Ts=0.7sec Vol (m^3)</th>
<th>Ts=1.0sec Vol (m^3)</th>
<th>Ts=1.3sec Vol (m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>24.0</td>
<td>88.2</td>
<td>185.9</td>
</tr>
<tr>
<td>Q</td>
<td>32.2</td>
<td>103.6</td>
<td>204.9</td>
</tr>
</tbody>
</table>

Note: See figure 4.

The defense space refers to the maximal range where the receiver can return the service before its second ground touching. The volume of the defense space for the two subjects in this study was calculated (Table 3). When the time spent in moving from the preparatory position to intercept the ball was 0.7s, the space for subject P was 24.0 m^3 and the space was 32.2 m^3 for subject Q. When the movement time was 1.3s, it was 185.9 m^3 for P and 204.9 m^3 for Q. It was indicated in Figure 4 that the volume of each subject’s defense space enlarged with the increase of defense time (Table 3). Besides, the result showed that with the same defense time, subject Q had larger defense space than subject P (Table 3), which suggested that subject Q moved faster and thus was able to intercept the service farther away from the preparatory position. When the movement time was 0.7s, both subject P and subject Q had defense space of irregular pattern (Fig. 4). Nevertheless, with the increase of the returning time, their defense space was shaped like a pie chart. This manifested that whichever direction the light indicated, the subjects performed well in defense.

Relationship between the placement and interception time

Table 4. The placement and the interception time

<table>
<thead>
<tr>
<th>Case</th>
<th>Subjects – win point</th>
<th>Vs (m/s)</th>
<th>Placement</th>
<th>Time(s)</th>
<th>Tf(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>P-No</td>
<td>25</td>
<td>(3) (7) (0)</td>
<td>0.907</td>
<td>-0.228</td>
</tr>
<tr>
<td>B</td>
<td>P-Yes</td>
<td>25</td>
<td>(0) (7) (0)</td>
<td>0.790</td>
<td>0.338</td>
</tr>
<tr>
<td>C</td>
<td>P-No</td>
<td>25</td>
<td>(3.5) (7) (0)</td>
<td>0.933</td>
<td>-0.432</td>
</tr>
<tr>
<td>D</td>
<td>Q-No</td>
<td>25</td>
<td>(3) (7) (0)</td>
<td>0.985</td>
<td>-0.273</td>
</tr>
<tr>
<td>E</td>
<td>Q-Yes</td>
<td>25</td>
<td>(0) (7) (0)</td>
<td>0.797</td>
<td>0.351</td>
</tr>
<tr>
<td>F</td>
<td>Q-No</td>
<td>25</td>
<td>(3.8) (7) (0)</td>
<td>0.989</td>
<td>-0.370</td>
</tr>
</tbody>
</table>

Note: contact position (0, -8, 2.2); the preparatory position of subjects P and Q(0,8.5,0); Tf: ahead of time (+) or behind the time (-)(See figure 5).

![Figure 5. The relationship between the striker’s contact position (0,-8,2.2) and the preparatory position of subjects P and Q (0,8.5,0)](image)

The striker’s contact position was hypothesized to be \( P_0 = (0, -8, 2.2)(\text{Fig.}5\text{b}) \) with the contact speed \( V_s=25\text{m/s} \). It was assumed that the placement position was \( (0, 7, 0) \), and the preparatory position of subject P was \( (0, 8.5, 0) \) (Table 4). The simulation result indicated that to intercept the ball, subject P had to spend no more than 0.790s, and subject P made it 0.338s ahead of time. However, if the placement was switched to the position \( (3, 7, 0) \); namely, it was moved 3m toward the X-axis (Fig.5a), it was found that subject P would be 0.228s behind the required time and unable to intercept the ball. Suppose the placement was located in the position \( (-3.5, 7, 0) \), subject P would be 0.432s behind the required time and fail to...
intercept the ball, either (Fig.5c).

Next, the situation of subject Q was explored. The striker’s contact speed was also assumed to be 25m/s (Fig.5d,5e,5f) with three different placements, one on the center, another on the left side, and the other on the right side (Table 4). The simulation result also demonstrated that it was easier for subject Q to intercept the ball if the placement was located in the area right in front of subject Q rather than on either side.

Simulation results of scoring placements in tennis singles

![Figure 6. Simulation result of subjects P and Q (placements in gray squares were scoring placements for the striker, placements in blue squares were nonscoring ones) (a) Trajectories for different placements when the contact position was (-4,-8,2.2) and the receiver was subject P (b) The three optimal placements when the receiver was subject P (Table 5. case a, b). (c) Trajectories for different placements when the contact position was (-4,-8,2.2) and the receiver was subject Q (d) The three optimal placements when the receiver was subject Q (Table 5. case c, d).](image)

Table 5. Relative parameters in simulating the contact position

<table>
<thead>
<tr>
<th>Case</th>
<th>Subjects</th>
<th>Vs(m/s)</th>
<th>X(m)</th>
<th>Y(m)</th>
<th>Z(m)</th>
<th>Preparatory Position</th>
<th>Scoring rate(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a,b</td>
<td>P</td>
<td>25</td>
<td>-4</td>
<td>-8.0</td>
<td>2.2</td>
<td>0</td>
<td>7.5</td>
</tr>
<tr>
<td>c,d</td>
<td>Q</td>
<td>25</td>
<td>4</td>
<td>-8.0</td>
<td>2.2</td>
<td>0</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Scoring rate =100% × (the number of scoring placements/100 placements)(See figure 6)

To conduct simulation on subject P and subject Q, we adopted the flying algorithm and the algorithm for the defense space of returning the service, taking into account the parameters listed in Table 5. The striker’s placements were sequenced with the computer, which contributed to the priority for returning (Fig. 6) When the striker’s contact position was \( P_0 = (-4, -8, 2.2) \) and \( V_s = 25 \text{ m/s} \) (Table 5), subject P moved from the preparatory position \((0,7,0)\) to return the ball (Fig 6a). It was found that the scoring rate for the striker was up to 39%.

The striker’s optimal placement was in the right forward of subject P’s initial position (Fig 6b). The same procedure with the above conditions was also conducted on subject Q (Table 5). It was found that the scoring rate for the striker was 38%. Those placements in the right forward of subject Q’s initial position, which would make it most difficult to return the ball (Fig.6d), and thus became the striker’s optimal placements.

As was shown from the examples of subject P and subject Q, a principle could be demonstrated. That is, the farther the placement is away from the preparatory position of the receiver, the more difficult it is for the receiver to return the service and the easier to lose a point. This phenomena supported the concept that an advantageous placement should be located on either side of the receiver, so that the receiver...
can’t return the ball in time and loses a point [4].

Simulation results of scoring placements in tennis doubles

Figure 7. Distribution of a single ball’s placement positions and the trajectories of optimal placements in tennis doubles (See table 6); the optimal placements are distributed on either side of the receiver.

Table 6. Parameters for contact positions and the preparatory position of receivers P and Q

<table>
<thead>
<tr>
<th>case</th>
<th>V(m/s)</th>
<th>X(m)</th>
<th>Y(m)</th>
<th>Z(m)</th>
<th>A-X</th>
<th>A-Y</th>
<th>A-Z</th>
<th>B-X</th>
<th>B-Y</th>
<th>B-Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>25</td>
<td>3</td>
<td>-9.0</td>
<td>2.0</td>
<td>-2</td>
<td>9</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>35</td>
<td>3</td>
<td>-9.0</td>
<td>2.0</td>
<td>-2</td>
<td>9</td>
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<td>2</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>25</td>
<td>0</td>
<td>-9.0</td>
<td>2.0</td>
<td>-2</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>35</td>
<td>0</td>
<td>-9.0</td>
<td>2.0</td>
<td>-2</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>8</td>
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<td>25</td>
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<td>-9.0</td>
<td>2.0</td>
<td>-2</td>
<td>5</td>
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<td>2</td>
<td>9</td>
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</tr>
<tr>
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<td>5</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: See figure 7

In simulating tennis doubles, the striker’s position was (25,3,-9.0) · Vs=25m/s, the preparatory position of subject P was (2.9,0 ) and for subject Q, the preparatory position was (2,5,0) (Table 6.). Scoring placements fell in the gray area, and the optimal ones were near the net of the right side (Fig 7a). When Vs was elevated to 35m/s, the optimal placement also fell near the net of the right side. With the increase of contact speed, four passing shots could be formed and touch the ground between the two receivers, which were out of the receivers’ reach and enabled the striker to score (Fig 7b).

Next, the striker’s contact position was moved to the middle P0 =(-0, -9, 2), with Vs being 25m/s(Fig 7.c) and 35m/s(Fig 7.d) respectively. It was found that the optimal placements fell near the net of the right side. The result also showed that seven passing shots would be formed and touch the ground between the two receivers (Fig 7d).
Finally, the striker’s contact position was moved to $P_0 = (-3, -9, 2)$. Simulation was conducted with $V_s$ being 25m/s (Fig 7.e) and 35m/s (Fig 7.f) respectively. The result indicated that the optimal placements were near the net of the right side. Three passing shots would touch the ground between the two receivers (Fig 7f).

Based on the above analysis, whether it is in singles or doubles, the farther the placement is away from the receiver, the easier for the striker to gain a point. The striker’s contact position and the receiver’s two side positions in returning the service was shaped like a V. The simulation conducted in this study evidenced that optimal placements were found outside of the V area, a phenomena called V effect. Consequently, the result confirms that our simulation system is capable of testifying the theory of V effect, and that the mathematical model designed in this study has practical value.

**Simulation of defense in singles**

<table>
<thead>
<tr>
<th>Case</th>
<th>$V_s$(m/s)</th>
<th>$X$(m)</th>
<th>$Y$(m)</th>
<th>$Z$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0-25</td>
<td>5</td>
<td>-7.5</td>
<td>1.2</td>
</tr>
<tr>
<td>b</td>
<td>0-25</td>
<td>0</td>
<td>-7.5</td>
<td>1.2</td>
</tr>
<tr>
<td>c</td>
<td>0-25</td>
<td>-5</td>
<td>-7.5</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note: See figure 8

**Table 8. contact speed and contact position in doubles**

<table>
<thead>
<tr>
<th>Case</th>
<th>$V_s$(m/s)</th>
<th>$X$(m)</th>
<th>$Y$(m)</th>
<th>$Z$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
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<td>5</td>
<td>-6</td>
<td>0.5</td>
</tr>
<tr>
<td>e</td>
<td>0-25</td>
<td>0</td>
<td>-6</td>
<td>0.5</td>
</tr>
<tr>
<td>f</td>
<td>0-25</td>
<td>-5</td>
<td>-6</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: See figure 8

Figure 8. The area bisecting effect (a,b,c) was found in the optimal defense of singles (See table 7); an isosceles triangle effect (d,e,f) was found in the optimal defense of doubles (See table 8).

The striker’s maximal $V_s$ was set to be 25m/s. Under this condition, simulation of 100 services was conducted on the defense and offense with three different contact positions (Table 7). The computer sequenced the service returns in the order of optimum and generated a priority for retruring. It was found that when the contact position was on the corner of the court’s left side, the optimal returning position of the receiver was on the corner of the court’s right side (Fig 8a). The two positions formed a diagonal line. Nevertheless, when the contact position was moved to the middle (0, 7.5, 1.2), the optimal defense space was in the middle part of the later half

The contact position and the returning position formed a bisecting line (Fig 8b). Next, when the contact position was on the corner of the court’s right side, the optimal returning position of the receiver was on the corner of the court’s left side. The two positions formed a diagonal line (Fig 8c). Based on the analysis of the above three cases, two principles could be induced: (1) the receiver’s preparatory position for returning the service changed with the variation of the striker’s contact position, (2) in singles, a diagonal line (Fig 8a, 8c) or a bisecting line (Fig 8b) would be formed between the striker’s contact position and the receiver’s optimal returning position.
Either of the two lines bisected the court, which we called area bisecting effect.

**Simulation of defense in doubles**

With Vs at the maximum of 25m/s, simulation of 25 services was conducted on the defense and offense with three different contact points (Table 8). The computer sequenced the service returns of receivers P and Q in the order of optimum and generated a priority for returning. It was found that when the contact point was (5,-6,0.5) of the left court, an isosceles triangle would be formed between the vertex angle of contact point and the base angles of receivers P and Q’s preparatory positions (Fig 8d). Likewise, when the contact point was (0,-6,0.5) of the middle part, an isosceles triangle was formed between the vertex angle of contact point and the base angles of receivers P and Q’s preparatory positions (Fig 8c). Next, when the contact point was (-5,6,0.5) of the right court, an isosceles triangle would be formed between the vertex angle of contact point and the base angles of receivers P and Q’s preparatory positions (Fig 8f).

The above analyses pointed to a common result. Wherever the contact point was located, an isosceles triangle would be formed between the vertex angle of the contact point and the base angle of each preparatory position. The area of the isosceles triangle was the optimal defense area for two receivers. We called it the isosceles triangle effect.

To testify the prevalence of the isosceles triangle effect, we conducted simulation at Vs elevated to 30m/s as well as declined to 20m/s. The result also showed that an isosceles triangle would be formed between the vertex angle of the contact point and the base angle of each preparatory position. The area of the isosceles triangle was the optimal defense space for the receivers. Obviously, to successfully return the service, the receivers should put emphasis on the isosceles triangle effect, which could make a great strategy.

**Conclusion**

A mathematic model was established in this study to simulate the serving and service returning in singles and doubles. It was discovered from the simulation results that three effects emerged: (1) Whether it is a tennis single or double, the farther the placement is away from the defense area of either side of the receiver, the more likely it is for the striker to score. This phenomena is called V effect, (2) in singles, two points, the contact position and the optimal defense position, forms a diagonal line or a bisecting line. Either of the two lines can evenly divide the court, which is called area bisecting effect, (3) in doubles, if the three points, the contact position and the preparatory position of each receiver, form an isosceles triangle, it is to the receivers’ advantage. This phenomena is called an isosceles triangle effect. In the future, further studies should be done on real competitions to prove the validity of the three effects. Also, it is expected that the algorithm designed in this study can be used to establish an offense and defense model of multiballs so that the mathematical model can assist tennis players in effective training.

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